```
> restart:

  with(linalg):
  with(LinearAlgebra):
  with(DifferentialGeometry):
  with(Tools):
  with(PDEtools, casesplit, declare):
  with(DEtools, gensys):
  interface(rtablesize=infinity):
```

```
> ################################################
> # NORMALISATION: May set f2=-g3^2*c/(2*g1), b=0 ##
> ################################################
```

```
> DGsetup([x,y,z,p], [c,g1,g3],
          Variete_groupe_coordonnees):
```

```
> declare(F(x,y,z,p));
```

$$F(x, y, z, p) \text{ will now be displayed as } F \quad (1)$$

```
> Fp := diff(F(x,y,z,p), p);
  Fpp := diff(F(x,y,z,p), p,p);
  Fxy := diff(F(x,y,z,p), x,y);
```

$$Fp := F_{p}$$

$$Fpp := F_{p, p}$$

$$Fxy := F_{x, y} \quad (2)$$

```
> ## INTRODUCE A BASIC INITIAL COFRAME

  ## L  := evalDG(dz-p*dx-F(x,y,z,p)*dy);
  ## M := evalDG(dp);
  ## N1 := evalDG(dx+Fp*dy);
  ## N2 := evalDG(Fpp*dy);
```

```
> ## INTRODUCE A BASIC INITIAL COFRAME

  L  := evalDG(dz-p*dx-(1/2)*p^2*dy);
  M := evalDG(dp);
  N1 := evalDG(dx+p*dy);
  N2 := evalDG(dy);
```

$$L := -p\, dx - \frac{p^{2}\, dy}{2} + dz$$

$$M := dp$$

$$N1 := dx + p\, dy$$

$$N2 := dy \quad (3)$$

```
> G := Matrix([ [c*g1,   0,  0, 0],
                [0, c, 0, 0],
                 [-c*g3, 0, g1, 0],
                 [-g3^2*c/(2*g1),  0,  g3, g1/c] ]);
```

$$G := \begin{bmatrix} c\,g1 & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ -c\,g3 & 0 & g1 & 0 \\ -\frac{g3^2\,c}{2\,g1} & 0 & g3 & \frac{g1}{c} \end{bmatrix} \quad (4)$$

```
> Ginverse := inverse(G);
```

$$Ginverse := \begin{bmatrix} \frac{1}{g1\,c} & 0 & 0 & 0 \\ 0 & \frac{1}{c} & 0 & 0 \\ \frac{g3}{g1^2} & 0 & \frac{1}{g1} & 0 \\ -\frac{c\,g3^2}{2\,g1^3} & 0 & -\frac{c\,g3}{g1^2} & \frac{c}{g1} \end{bmatrix} \quad (5)$$

```
> dG:=evalDG(ExteriorDerivative(G));
```

$$dG := \begin{bmatrix} g1\,dc + c\,dg1 & 0\,dx & 0\,dx & 0\,dx \\ 0\,dx & dc & 0\,dx & 0\,dx \\ -g3\,dc - c\,dg3 & 0\,dx & dg1 & 0\,dx \\ -\frac{g3^2\,dc}{2\,g1} + \frac{g3^2\,c\,dg1}{2\,g1^2} - \frac{g3\,c\,dg3}{g1} & 0\,dx & dg3 & -\frac{g1\,dc}{c^2} + \frac{dg1}{c} \end{bmatrix} \quad (6)$$

```
> MC := evalDG(dG.Ginverse);
```

$$MC := \begin{bmatrix} \frac{dc}{c} + \frac{dg1}{g1} & 0\,dx & 0\,dx & 0\,dx \\ 0\,dx & \frac{dc}{c} & 0\,dx & 0\,dx \\ -\frac{g3\,dc}{g1\,c} + \frac{g3\,dg1}{g1^2} - \frac{dg3}{g1} & 0\,dx & \frac{dg1}{g1} & 0\,dx \\ 0\,dx & 0\,dx & \frac{g3\,dc}{g1\,c} - \frac{g3\,dg1}{g1^2} + \frac{dg3}{g1} & -\frac{dc}{c} + \frac{dg1}{g1} \end{bmatrix} \quad (7)$$

```
> oldcoframe := Vector([L,M,N1,N2]);
```

$$oldcoframe := \begin{bmatrix} -p\,dx - \frac{p^2\,dy}{2} + dz \\ dp \\ dx + p\,dy \\ dy \end{bmatrix} \tag{8}$$

```
> newcoframe := evalDG(G.oldcoframe);
```

$$newcoframe := \begin{bmatrix} -c\,g1\,p\,dx - \frac{c\,g1\,p^2\,dy}{2} + c\,g1\,dz \\ c\,dp \\ (c\,g3\,p + g1)\,dx + \left(g1\,p + \frac{1}{2}\,c\,g3\,p^2\right)dy - c\,g3\,dz \\ \frac{g3\,(c\,g3\,p + 2\,g1)\,dx}{2\,g1} + \frac{\left(c^2\,g3^2\,p^2 + 4\,c\,g1\,g3\,p + 4\,g1^2\right)dy}{4\,c\,g1} - \frac{g3^2\,c\,dz}{2\,g1} \end{bmatrix} \tag{9}$$

```
> AA := MC[3,3];
  BB := MC[2,2];
  # CC := MC[2,1];
  DD := MC[3,1];
  # SS := MC[4,1];
  # TT := MC[4,3];
  L1 := newcoframe[1];
  M1 := newcoframe[2];
  N1l := newcoframe[3];
  N2l := newcoframe[4];
```

$$AA := \frac{dg1}{g1}$$

$$BB := \frac{dc}{c}$$

$$DD := -\frac{g3\,dc}{g1\,c} + \frac{g3\,dg1}{g1^2} - \frac{dg3}{g1}$$

$$L1 := -c\,g1\,p\,dx - \frac{c\,g1\,p^2\,dy}{2} + c\,g1\,dz$$

$$M1 := c\,dp$$

$$N1l := (c\,g3\,p + g1)\,dx + \left(g1\,p + \frac{1}{2}\,c\,g3\,p^2\right)dy - c\,g3\,dz$$

$$N2l := \frac{g3\,(c\,g3\,p + 2\,g1)\,dx}{2\,g1} + \frac{\left(c^2\,g3^2\,p^2 + 4\,c\,g1\,g3\,p + 4\,g1^2\right)dy}{4\,c\,g1} - \frac{g3^2\,c\,dz}{2\,g1} \tag{10}$$

```
> coframeloop1 := FrameData([AA,BB,DD,Ll,Ml,N1l,N2l],coframe1):
```

```
> DGsetup(coframeloop1, [E],
[aa,bb,dd,lambda,mu,nu[1],nu[2]],  verbose);
```

The following coordinates have been protected:

$$[x, y, z, p, c, g1, g3]$$

The following vector fields have been defined and protected:

$$[E1, E2, E3, E4, E5, E6, E7]$$

The following differential 1-forms have been defined and protected:

$$[aa, bb, dd, \lambda, \mu, \nu_1, \nu_2]$$

frame name: coframe1 **(11)**

```
> dlambda:=evalDG(ExteriorDerivative(lambda))
```

$$dlambda := aa \wedge \lambda + bb \wedge \lambda + \frac{g3\,\lambda}{g1} \wedge \mu - \mu \wedge \nu_1 \quad \textbf{(12)}$$

```
> dmu := evalDG(ExteriorDerivative(mu));
```

$$dmu := bb \wedge \mu \quad \textbf{(13)}$$

```
> dnu1 := evalDG(ExteriorDerivative(nu[1]));
```

$$dnu1 := aa \wedge \nu_1 + dd \wedge \lambda - \frac{g3^2\,\lambda}{2\,g1^2} \wedge \mu + \mu \wedge \nu_2 \quad \textbf{(14)}$$

```
> dnu2 := evalDG(ExteriorDerivative(nu[2]));
```

$$dnu2 := aa \wedge \nu_2 - bb \wedge \nu_2 - dd \wedge \nu_1 - \frac{g3^2\,\mu}{2\,g1^2} \wedge \nu_1 + \frac{g3\,\mu}{g1} \wedge \nu_2 \quad \textbf{(15)}$$

```
> ##################################################################
##############
> ### VERY CRUCIAL STEP: DEFINE DUAL VECTOR FIELD OF THE (COFRAME :=
COBASIS)  ####
> ### MAPLE DOES NOT ALLOW ABSORPTION DIRECTLY BY 1-FORMS
         ####
> ### ABSORPTION HAVE TO BE DONE BY (VECTOR FIELDS := DB
         ####
> ##################################################################
##############
```

```
> cobasis := [AA,BB,DD,Ll,Ml,N1l,N2l];
```

$$cobasis := \left[\frac{dg1}{g1}, \frac{dc}{c}, -\frac{g3\,dc}{g1\,c} + \frac{g3\,dg1}{g1^2} - \frac{dg3}{g1}, -c\,g1\,p\,dx - \frac{c\,g1\,p^2\,dy}{2} + c\,g1\,dz, \right. \quad \textbf{(16)}$$

$$c\,dp,\ (c\,g3\,p+g1)\,dx+\left(g1\,p+\frac{1}{2}\,c\,g3\,p^2\right)dy-c\,g3\,dz,\ \frac{g3\,(c\,g3\,p+2\,g1)\,dx}{2\,g1}+\frac{(c^2\,g3^2\,p^2+4\,c\,g1\,g3\,p+4\,g1^2)\,dy}{4\,c\,g1}-\frac{g3^2\,c\,dz}{2\,g1}\Big]$$

```
> DB:=DualBasis(cobasis)
```

$$DB := \left[g1\,D\_g1+g3\,D\_g3,\ c\,D\_c-g3\,D\_g3,\ \text{-}g1\,D\_g3,\ \frac{g3\,(c\,g3\,p+2\,g1)\,D\_x}{2\,g1^3}-\frac{c\,g3^2\,D\_y}{2\,g1^3}+\frac{(c\,g3\,p+2\,g1)^2\,D\_z}{4\,c\,g1^3},\ \frac{D\_p}{c},\ \frac{(c\,g3\,p+g1)\,D\_x}{g1^2}-\frac{c\,g3\,D\_y}{g1^2}+\frac{(c\,g3\,p+2\,g1)\,p\,D\_z}{2\,g1^2},\ \text{-}\frac{c\,p\,D\_x}{g1}+\frac{c\,D\_y}{g1}-\frac{c\,p^2\,D\_z}{2\,g1}\right] \quad \textbf{(17)}$$

```
> ############################################################
> ### INTRODUCE NEW VARIBLES FOR ABSORPTION               #####
> ############################################################
```

```
> variables := [ seq(A[i], i=1..4),
                 seq(B[i], i=1..4),
                 seq(D[i], i=1..4)
                    ];

  nops(variables);
```

$$variables := [A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, \mathrm{D}_1, \mathrm{D}_2, \mathrm{D}_3, \mathrm{D}_4]$$

$$12 \quad \textbf{(18)}$$

```
> ####################################################
> ## ABSORPTION PROCESS: DO IT VIA THE VECTOR FIELD! ##
> ####################################################
```

```
> Daa := DB[1];
  Dbb := DB[2];
  Ddd := DB[3];
  Dlambda := A[1]*DB[1]+B[1]*DB[2]+D[1]*DB[3]+DB[4];
  Dmu :=     A[2]*DB[1]+B[2]*DB[2]+D[2]*DB[3]+DB[5];
  Dnu1 :=    A[3]*DB[1]+B[3]*DB[2]+D[3]*DB[3]+DB[6];
  Dnu2 :=    A[4]*DB[1]+B[4]*DB[2]+D[4]*DB[3]+DB[7];
```

$$Daa := g1\,D\_g1+g3\,D\_g3$$

$$Dbb := c\,D\_c-g3\,D\_g3$$

$$Ddd := \text{-}g1\,D\_g3$$

$$Dlambda := A_1 \, g1 \, D\_g1 + g3 \, D\_g3 + B_1 \, c \, D\_c - g3 \, D\_g3 + \mathrm{D}_1 \, \text{-}g1 \, D\_g3 + \frac{g3 \, (c \, g3 \, p + 2 \, g1) \, D\_x}{2 \, g1^3} - \frac{c \, g3^2 \, D\_y}{2 \, g1^3} + \frac{(c \, g3 \, p + 2 \, g1)^2 \, D\_z}{4 \, c \, g1^3}$$

$$Dmu := A_2 \, g1 \, D\_g1 + g3 \, D\_g3 + B_2 \, c \, D\_c - g3 \, D\_g3 + \mathrm{D}_2 \, \text{-}g1 \, D\_g3 + \frac{D\_p}{c}$$

$$Dnu1 := A_3 \, g1 \, D\_g1 + g3 \, D\_g3 + B_3 \, c \, D\_c - g3 \, D\_g3 + \mathrm{D}_3 \, \text{-}g1 \, D\_g3 + \frac{(c \, g3 \, p + g1) \, D\_x}{g1^2} - \frac{c \, g3 \, D\_y}{g1^2} + \frac{(c \, g3 \, p + 2 \, g1) \, p \, D\_z}{2 \, g1^2}$$

$$Dnu2 := A_4 \, g1 \, D\_g1 + g3 \, D\_g3 + B_4 \, c \, D\_c - g3 \, D\_g3 + \mathrm{D}_4 \, \text{-}g1 \, D\_g3 + \text{-}\frac{c \, p \, D\_x}{g1} + \frac{c \, D\_y}{g1} - \frac{c \, p^2 \, D\_z}{2 \, g1} \qquad \textbf{(19)}$$

```
> ######################################################
> ## GET (UNMODIFIED) TENSORS BY INTERIOR PRODUCT !! ##
> ######################################################
```

```
> LTensor := [
  Hook([DB[4],DB[5]],ExteriorDerivative(Ll)),
  Hook([DB[4],DB[6]],ExteriorDerivative(Ll)),
  Hook([DB[4],DB[7]],ExteriorDerivative(Ll)),
  Hook([DB[5],DB[6]],ExteriorDerivative(Ll)),
  Hook([DB[5],DB[7]],ExteriorDerivative(Ll)),
  Hook([DB[6],DB[7]],ExteriorDerivative(Ll))];

  MTensor :=[
  Hook([DB[4],DB[5]],ExteriorDerivative(Ml)),
  Hook([DB[4],DB[6]],ExteriorDerivative(Ml)),
  Hook([DB[4],DB[7]],ExteriorDerivative(Ml)),
  Hook([DB[5],DB[6]],ExteriorDerivative(Ml)),
  Hook([DB[5],DB[7]],ExteriorDerivative(Ml)),
  Hook([DB[6],DB[7]],ExteriorDerivative(Ml))];

  N1Tensor :=[
  Hook([DB[4],DB[5]],ExteriorDerivative(N1l)),
  Hook([DB[4],DB[6]],ExteriorDerivative(N1l)),
  Hook([DB[4],DB[7]],ExteriorDerivative(N1l)),
  Hook([DB[5],DB[6]],ExteriorDerivative(N1l)),
  Hook([DB[5],DB[7]],ExteriorDerivative(N1l)),
  Hook([DB[6],DB[7]],ExteriorDerivative(N1l))];

  N2Tensor:= [
  Hook([DB[4],DB[5]],ExteriorDerivative(N2l)),
  Hook([DB[4],DB[6]],ExteriorDerivative(N2l)),
  Hook([DB[4],DB[7]],ExteriorDerivative(N2l)),
  Hook([DB[5],DB[6]],ExteriorDerivative(N2l)),
  Hook([DB[5],DB[7]],ExteriorDerivative(N2l)),
  Hook([DB[6],DB[7]],ExteriorDerivative(N2l))];
```

$$LTensor := \left[\frac{g3}{g1}, 0, 0, -1, 0, 0\right]$$

$$MTensor := [0, 0, 0, 0, 0, 0]$$

$$N1Tensor := \left[-\frac{g3^2}{2\,g1^2}, 0, 0, 0, 1, 0\right]$$

$$N2Tensor := \left[0, 0, 0, -\frac{g3^2}{2\,g1^2}, \frac{g3}{g1}, 0\right] \quad \textbf{(20)}$$

```
> ###################################################################
> ## GET MODIFIED TENSORS FOR ABSORPTION BY INTERIOR PRODUCT !! ##
> ###################################################################
```

```
> LTensorm := [Hook([Dlambda,Dmu],ExteriorDerivative(Ll)),
  Hook([Dlambda,Dnu1],ExteriorDerivative(Ll)),
  Hook([Dlambda,Dnu2],ExteriorDerivative(Ll)),
  Hook([Dmu,Dnu1],ExteriorDerivative(Ll)),
  Hook([Dmu,Dnu2],ExteriorDerivative(Ll)),
  Hook([Dnu1,Dnu2],ExteriorDerivative(Ll))];

  MTensorm := [Hook([Dlambda,Dmu],ExteriorDerivative(Ml)),
  Hook([Dlambda,Dnu1],ExteriorDerivative(Ml)),
  Hook([Dlambda,Dnu2],ExteriorDerivative(Ml)),
  Hook([Dmu,Dnu1],ExteriorDerivative(Ml)),
  Hook([Dmu,Dnu2],ExteriorDerivative(Ml)),
  Hook([Dnu1,Dnu2],ExteriorDerivative(Ml))];

  N1Tensorm := [Hook([Dlambda,Dmu],ExteriorDerivative(N1l)),
  Hook([Dlambda,Dnu1],ExteriorDerivative(N1l)),
  Hook([Dlambda,Dnu2],ExteriorDerivative(N1l)),
  Hook([Dmu,Dnu1],ExteriorDerivative(N1l)),
  Hook([Dmu,Dnu2],ExteriorDerivative(N1l)),
  Hook([Dnu1,Dnu2],ExteriorDerivative(N1l))];

  N2Tensorm := [Hook([Dlambda,Dmu],ExteriorDerivative(N2l)),
  Hook([Dlambda,Dnu1],ExteriorDerivative(N2l)),
  Hook([Dlambda,Dnu2],ExteriorDerivative(N2l)),
  Hook([Dmu,Dnu1],ExteriorDerivative(N2l)),
  Hook([Dmu,Dnu2],ExteriorDerivative(N2l)),
  Hook([Dnu1,Dnu2],ExteriorDerivative(N2l))];
```

$$LTensorm := \left[-\frac{A_2\,g1 + B_2\,g1 - g3}{g1}, -A_3 - B_3, -A_4 - B_4, -1, 0, 0\right]$$

$$MTensorm := [B_1, 0, 0, -B_3, -B_4, 0]$$

$$N1Tensorm := \left[-\frac{2\,g1^2\,D_2 + g3^2}{2\,g1^2}, A_1 - D_3, -D_4, A_2, 1, -A_4\right]$$

$$N2Tensorm := \left[0, -D_1, A_1 - B_1, -\frac{2\,g1^2\,D_2 + g3^2}{2\,g1^2}, \frac{A_2\,g1 - B_2\,g1 + g3}{g1}, A_3 - B_3 + D_4\right] \quad \textbf{(21)}$$

```
> ##################################################################
> ## Combine LTensor, MTensor, N1Tensor, N2Tensor AS ONE ARRAY##
> ##################################################################
```

```
> Tensors := [op(LTensor), op(MTensor), op(N1Tensor), op(N2Tensor)]
```

$$Tensors := \left[\frac{g3}{g1}, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, -\frac{g3^2}{2\,g1^2}, 0, 0, 0, 1, 0, 0, 0, 0, -\frac{g3^2}{2\,g1^2}, \frac{g3}{g1}, 0\right] \quad (22)$$

```
> #######################################################################
####
> ## Combine LTensorm, MTensorm, N1Tensorm, N2Tensorm AS ONE ARRAY
 ##
> ## FIND OUT THE NUMBER OF EQUATIONS (EVEN NON-CONSISTENT)
 ##
> ## WRITE EXPLICIT ABSORPTION EQUATIONS
 ##
> #######################################################################
####
```

```
> eqabsorb := [op(LTensorm), op(MTensorm), op(N1Tensorm), op
(N2Tensorm)]
```

$$eqabsorb := \left[-\frac{A_2\,g1 + B_2\,g1 - g3}{g1}, -A_3 - B_3, -A_4 - B_4, -1, 0, 0, B_1, 0, 0, -B_3, -B_4, 0, -\frac{2\,g1^2\,\mathrm{D}_2 + g3^2}{2\,g1^2}, A_1 - \mathrm{D}_3, -\mathrm{D}_4, A_2, 1, -A_4, 0, -\mathrm{D}_1, A_1 - B_1, -\frac{2\,g1^2\,\mathrm{D}_2 + g3^2}{2\,g1^2}, \frac{A_2\,g1 - B_2\,g1 + g3}{g1}, A_3 - B_3 + \mathrm{D}_4\right] \quad (23)$$

```
> nops(eqabsorb);
```

$$24 \quad (24)$$

```
> for i from 1 to nops(eqabsorb) do
expand(eqabsorb[i]=0)
od;
```

$$-A_2 - B_2 + \frac{g3}{g1} = 0$$

$$-A_3 - B_3 = 0$$

$$-A_4 - B_4 = 0$$

$$-1 = 0$$

$$0 = 0$$
$$0 = 0$$
$$B_1 = 0$$
$$0 = 0$$
$$0 = 0$$
$$-B_3 = 0$$
$$-B_4 = 0$$
$$0 = 0$$
$$-\mathrm{D}_2 - \frac{g3^2}{2\, g1^2} = 0$$
$$A_1 - \mathrm{D}_3 = 0$$
$$-\mathrm{D}_4 = 0$$
$$A_2 = 0$$
$$1 = 0$$
$$-A_4 = 0$$
$$0 = 0$$
$$-\mathrm{D}_1 = 0$$
$$A_1 - B_1 = 0$$
$$-\mathrm{D}_2 - \frac{g3^2}{2\, g1^2} = 0$$
$$A_2 - B_2 + \frac{g3}{g1} = 0$$
$$A_3 - B_3 + \mathrm{D}_4 = 0 \qquad \textbf{(25)}$$

```
> ##########################################################################
> ## PROF MERKER'S METHOD: FIND OUT WHICH VARIABLES NEED TO BE NORMALISED   ###
> ##########################################################################
```

```
> ABS_Eqst := transpose(genmatrix(eqabsorb, variables));
```

$ABS\_Eqst := [[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],$ **(26)**

$[-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0],$

[0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1],
[0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0],
[−1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0],
[0, −1, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1],
[0, 0, −1, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, −1, 0, 0, 0, 0, 0, 0, 0, 0, 1]]

```
> noyau := kernel(ABS_Eqst)
```

noyau := {[−1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −2 0 0 0 0 0 0 1 0], **(27)**
[0 0 −1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 1 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0],
[0 1 0 0 0 0 0 0 0 −2 0 0 0 0 1 0 0 0 0 0 0 0 0 1]}

```
> for ii from 1 to nops(noyau) do
  LC[ii] := convert(op(ii,noyau), list, nested=false)
  od;
```

$LC_1 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]$

$LC_2 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0]$

$LC_3 := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$

$LC_4 := [-1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -2, 0, 0, 0, 0, 0, 0, 1, 0]$

$LC_5 := [0, 1, 0, 0, 0, 0, 0, 0, 0, -2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1]$

$LC_6 := [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

$LC_7 := [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

$$LC_8 := [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$LC_9 := [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$LC_{10} := [0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$$
$$LC_{11} := [0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]$$
$$LC_{12} := [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad \textbf{(28)}$$

```
> Tensorv := convert(Tensors,Vector);
```

(29)

$$Tensorv := \begin{bmatrix} \frac{g3}{g1} \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{g3^2}{2\, g1^2} \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{g3^2}{2\, g1^2} \\ \frac{g3}{g1} \\ 0 \end{bmatrix} \quad \textbf{(29)}$$

```
> for i from 1 to nops(noyau) do
  factor(simplify(convert(LC[i],Vector[row]).Tensorv))
  od;
```

$$1$$
$$0$$
$$0$$
$$0$$

$$\begin{array}{c} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \quad (30)$$

```
> ###########################################################
> ### NOW THERE ARE NO MORE VARIABLES TO BE NORMALISED     ###
> ### PROCEED DIRECTLY TO E-STRUCTURE                      ###
> ### ABSORPTION BY INDIRECT METHODS                       ###
> ###########################################################
```

```
> ABS_Eqs := genmatrix(eqabsorb, variables)
```

(31)

$$ABS_Eqs := \left[\begin{array}{cccccccccccc}
0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1
\end{array}\right] \quad \textbf{(31)}$$

```
> ##
> ## SET UP THE MATRIX THE DESCRIBES THE EQUATION OF ABSORPTION
> ##
```

```
> Total := augment(ABS_Eqs, Tensors)
```

$$
Total := \left[\begin{array}{ccccccccccccc}
0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{g3^2}{2\,g1^2} \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{g3^2}{2\,g1^2} \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right] \qquad \textbf{(32)}
$$

```
> ##
> ## SOLVING NAIVELY BY GAUSS ELIMINATION
> ##
```

```
> Totale := gausselim(Total)
```

$$
Totale := \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{g3^2}{2\,g1^2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \qquad (33)
$$

```
> ##
> ## NEXT: TO REMOVE THE INCONSISTENCIES IN LINEAR EQUATION BY
  REMOVING LAST FEW ROWS
> ## M1 IS NOT A Matrix, PROCEED WITH CONVERSION
> ##

> M1 := convert(Totale, Matrix)
```

$$
M1 := \left[\begin{array}{ccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{g3^2}{2\,g1^2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \quad (34)
$$

```
> type(M1,Matrix)
```

true **(35)**

```
> ##
> ## DELETE REDUNDANT ROWS
> ##

> M1coh:=DeleteRow(M1,13..24)
```

$$M1coh := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{g3}{g1} \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\frac{g3^2}{2\,g1^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \quad \textbf{(36)}$$

```
> ##
> ## SOLVE ABSORPTION EQUATION
> ##
```

```
> ABSOL := LinearSolve(M1coh)
```

$$ABSOL := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\frac{g3}{g1} \\ 0 \\ 0 \\ 0 \\ \frac{g3^2}{2\,g1^2} \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(37)}$$

```
> whattype(ABSOL)
```

(38)

$$Vector_{column} \tag{38}$$

```
> ABSOLa:= convert(ABSOL,Array)
```

$$ABSOLa := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{g3}{g1} & 0 & 0 & 0 & \frac{g3^2}{2\,g1^2} & 0 & 0 \end{bmatrix} \tag{39}$$

```
> ABSOLa[6]
```

$$-\frac{g3}{g1} \tag{40}$$

```
> ##
> ## PROCEED WITH ABSORPTION BY VECTOR FIELDS
> ##
```

```
> ADaa  := DB[1];
  ADbb  := DB[2];
  ADdd  := DB[3];
  ADlambda := -ABSOLa[1]*DB[1]-ABSOLa[5]*DB[2]-ABSOLa[9]*DB[3]+DB[4];
  ADmu :=      -ABSOLa[2]*DB[1]-ABSOLa[6]*DB[2]-ABSOLa[10]*DB[3]+DB[5]
  ;
  ADnu1  :=    -ABSOLa[3]*DB[1]-ABSOLa[7]*DB[2]-ABSOLa[11]*DB[3]+DB[6]
  ;
  ADnu2  :=    -ABSOLa[4]*DB[1]-ABSOLa[8]*DB[2]-ABSOLa[12]*DB[3]+DB[7]
  ;
```

$$ADaa := g1\,D\_g1 + g3\,D\_g3$$

$$ADbb := c\,D\_c - g3\,D\_g3$$

$$ADdd := -g1\,D\_g3$$

$$ADlambda := \frac{g3\,(c\,g3\,p + 2\,g1)\,D\_x}{2\,g1^3} - \frac{c\,g3^2\,D\_y}{2\,g1^3} + \frac{(c\,g3\,p + 2\,g1)^2\,D\_z}{4\,c\,g1^3}$$

$$ADmu := \frac{g3\,c\,D\_c - g3\,D\_g3}{g1} - \frac{g3^2\,-g1\,D\_g3}{2\,g1^2} + \frac{D\_p}{c}$$

$$ADnu1 := \frac{(c\,g3\,p + g1)\,D\_x}{g1^2} - \frac{c\,g3\,D\_y}{g1^2} + \frac{(c\,g3\,p + 2\,g1)\,p\,D\_z}{2\,g1^2}$$

$$ADnu2 := -\frac{c\,p\,D\_x}{g1} + \frac{c\,D\_y}{g1} - \frac{c\,p^2\,D\_z}{2\,g1} \tag{41}$$

```
> ################################################
> ## CONVERSION BACK TO THE 1-FORMS            ##
> ## FIRST DEFINE A FRAME, THEN FIND ITS DUAL    ##
> ################################################
```

```
> Sfinal := [ADaa, ADbb, ADdd, ADlambda, ADmu, ADnu1, ADnu2]
```

$$Sfinal := \left[g1\,D\_g1 + g3\,D\_g3,\ c\,D\_c - g3\,D\_g3,\ -g1\,D\_g3,\ \frac{g3\,(c\,g3\,p + 2\,g1)\,D\_x}{2\,g1^3} \right. \tag{42}$$

$$- \frac{c\, g3^2\, D\_y}{2\, g1^3} + \frac{(c\, g3\, p + 2\, g1)^2\, D\_z}{4\, c\, g1^3}, \frac{g3\, c\, D\_c - g3\, D\_g3}{g1} - \frac{g3^2\, -g1\, D\_g3}{2\, g1^2}$$

$$+ \frac{D\_p}{c}, \frac{(c\, g3\, p + g1)\, D\_x}{g1^2} - \frac{c\, g3\, D\_y}{g1^2} + \frac{(c\, g3\, p + 2\, g1)\, p\, D\_z}{2\, g1^2}, -\frac{c\, p\, D\_x}{g1}$$

$$+ \frac{c\, D\_y}{g1} - \frac{c\, p^2\, D\_z}{2\, g1} \Bigg]$$

```
> Bfinal := evalDG(DualBasis(Sfinal))
```

$$Bfinal := \left[\frac{dg1}{g1}, -\frac{g3\, c\, dp}{g1} + \frac{dc}{c}, \frac{g3^2\, c\, dp}{2\, g1^2} - \frac{g3\, dc}{g1\, c} + \frac{g3\, dg1}{g1^2} - \frac{dg3}{g1}, -c\, g1\, p\, dx \right. \tag{43}$$

$$- \frac{c\, g1\, p^2\, dy}{2} + c\, g1\, dz, c\, dp, (c\, g3\, p + g1)\, dx + \frac{(c\, g3\, p + 2\, g1)\, p\, dy}{2} - c\, g3\, dz,$$

$$\left. \frac{g3\, (c\, g3\, p + 2\, g1)\, dx}{2\, g1} + \frac{(c\, g3\, p + 2\, g1)^2\, dy}{4\, g1\, c} - \frac{g3^2\, c\, dz}{2\, g1} \right]$$

```
> ESTRUCTURE := FrameData(Bfinal, final)
```

$$ESTRUCTURE := [d\,\Theta 1 = 0, d\,\Theta 2 = \Theta 3 \wedge \Theta 5, d\,\Theta 3 = -\Theta 2 \wedge \Theta 3, d\,\Theta 4 = \Theta 1 \wedge \Theta 4 \tag{44}$$

$$+ \Theta 2 \wedge \Theta 4 - \Theta 5 \wedge \Theta 6, d\,\Theta 5 = \Theta 2 \wedge \Theta 5, d\,\Theta 6 = \Theta 1 \wedge \Theta 6 + \Theta 3 \wedge \Theta 4 + \Theta 5 \wedge \Theta 7,$$

$$d\,\Theta 7 = \Theta 1 \wedge \Theta 7 - \Theta 2 \wedge \Theta 7 - \Theta 3 \wedge \Theta 6]$$